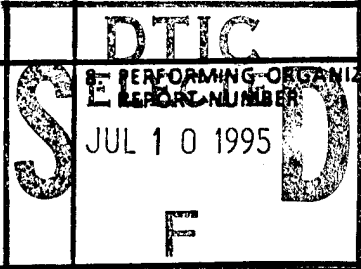| 1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE April 1995	3. REPORT TYPE AND DATES COVERED Final 1 Sep 91 – 28 Feb 95

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13. ABSTRACT (Maximum 200 words)

The dynamics of mistuned cyclic systems with special reference to strongly coupled bladed disk assemblies has been investigated. The analysis has utilized ideas from group representation theory, bifurcation theory, singular perturbation theory and modal analysis techniques. The general analysis methodology developed herein is applicable any disk attached with a set of n blades which are strongly coupled cyclically, and mistuning or variations can arise in any of the system parameters. In particular the study provides qualitative and more importantly, quantitative information in the form of uniformly valid asymptotic expansions for the eigen-frequencies and the modal vectors of the structure. These expansions are used to describe the phenomenon of eigenvalue veering, modal rotations, and other manifestations of the sensitive dependence of eigenfunctions on system parameters lead to modal bifurcations in the forced response of mistuned cyclic systems. Since this approach is general and systematic, the methodology developed is also extended and applied to other discrete and continuous structures as well.

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Structural Dynamics of Mistuned Spatially Periodic Mechanical Systems.

This final report documents the work performed at Southern University in close collaboration with Purdue University during the period of September 1991 through february 1995. The initial ARO Contract (#DAAL 03-91-G-0247) was for three years but a request for no-cost extension was granted for six months.

The objectives of the proposed research were to:

(i) Gain a fundamental understanding of how and why periodically configured mechanical and structural systems, (in particular bladed disk assemblies) with cyclic symmetry and nominally identical substructures can display non-uniform forced response amplitudes of vibration when subjected to small but random parameter perturbations that are often within the component manufacturing tolerances.

(ii) Develop an analytical method to carry out an in-depth investigation into the dynamics of mistuned n degree-of-freedom cyclic systems with special reference to bladed disk assemblies.

(iii) Develop a computer code to be able to perform parametric studies for the mistuned n degree-of-freedom linear cyclic systems.

(iv) Generalize the linear cyclic system models to include nonlinear coupling effects, and study the nonlinear system response for small motions.

(v) Apply the methodology to investigate the dynamics of continuous structures including thin elastic rings and plates with small mass and dimensional nonuniformities.

Status:

The work associated with the principal objectives of the project is completed and is included in this final report. It is established that uneven vibration amplitudes arise principally by the destruction of cyclic symmetry by some small perturbations usually within the component manufacturing tolerances. Such perturbations first split some of the eigenvalues degeneracies inherent in all cyclic systems. This splitting in turn gives rise to modal bifurcation phenomenon. Particular forms of the the modal bifurcations lead to uneven vibration amplitude, without mode localization being present. Ideas from group theory, bifurcation theory and singular perturbation theory are combined to provide a through analysis of general n-degree-of-freedom mistuned cyclic systems. The resulting asymptotically valid algebraic expansions for the eigenvalues and the eigenvectors have been used to study the forced response for any given excitation force.

Since this approach is general and systematic, the methodology developed here is also extended and applied to other discrete and continuous structures as well. The procedure for detecting a Priori which degenerate eigenvalue pairs will split under given parameter perturbations has been formalized by use of finite group representation theory and is presented in Chapter 2 and 3. The procedure for accurately unfolding the singularities induced by the splitting of the double modes, asymptotically valid algebraic expansions for eigenvalues and eigenfunctions, and expressions for forced amplitudes of vibration of blades, as formalized by the use of singular perturbation analysis technique combined with modal analysis, is included in Chapter 5. The general procedure for detecting singular points of a dynamical system and the use of singular perturbation methodology for a model problem is described in Chapter 4. This discussion also includes the ideas of modal sensitivity and the eigenvector rotations as criteria for describing the behavior of perturbed systems. The dynamics of perturbed continuous structures such as a rotating string and vibrating membrane are presented in Chapter 6. A computer code based on the singular perturbation analysis described in Chapter 5, and suitable for performing parametric studies with a linear cyclic model (n degree-of-freedom bladed disk assembly) is included in Appendix H. Generalization of the linear cyclic system models to include nonlinear coupling stiffness effects, and a study of the nonlinear system response for small motions are included in Appendix F.

Publications

Eight papers and a computer code have been developed from this work. The first, second and third papers are essentially the same as described in the main body of this report. Papers 1 and 3 are included in Appendix G. The papers from 4 to 7 are given in Appendix F. All the results generated in the eighth paper (in preparation) have been obtained from the computer code given in Appendix H.

1. Happawana, G.S., Bajaj, A.K. and Nwokah, O.D.I., 1993 "Modal Analysis and Forced Response of Coupled Mistuned Cyclic Systems: A Singular Perturbation Approach", ASME Paper 93-GT-266, Proceedings of the ASME IGII Conference, May 24-27, Cincinnati, Ohio.

2. Happawana, G.S., Bajaj, A.K., Nwokah, O.D.I. and Azene, M., 1994, "Free and Forced Response of Mistuned Linear Cyclic Systems: A Singular Perturbation Approach", Journal of Sound and Vibration. Submitted (being revised).

3. Happawana, G.S., Bajaj, A.K. and Nwokah, O.D.I., 1993, "Eigenvalue Veering, Perturbed Bifurcation Theory and Modal Analysis of Mistuned Linear Systems", Proceedings of the 14th ASME Biennial Conference on Mechanical Vibration and Noise, Vol. 59, pp. 1-13, September 19-22, Albuquerque, NM.

4. Bajaj, A.K., Samaranayake, S. and Nwokah, O.D.I., 1994, "Dynamic Response and Chaos in Resonantly Excited Structures with Cyclic Symmetry", Nonlinearity and Chaos in Engineering Dynamics, Eds. J.M.T. Thompson, S. Bishop, John Wiley, NY.

5. Samaranayake, S., Bajaj, A.K. and Nwokah, O.D.I., 1993, "Resonant Vibrations in Weakly Coupled Nonlinear Structures with Cyclic Symmetry", Proceedings of the 14th ASME Conference on Mechanical Vibration and Noise, September 19-22, Albuquerque, NM.

6. Samaranayake, S. Bajaj, A.K. and Nwokah, O.D.I., 1995, "Amplitude Modulated Dynamics and Bifurcations in the Resonant Response of a Structure with Cyclic Symmetry", Acta Mechanica, Vol. 109, pp. 101-125.

7. Happawana, G.S., Bajaj, A.K. and Azene, M., 1995, "An Analytical Solution to Non-Similar Normal Modes in a Strongly Nonlinear Discrete System", Journal of Sound and Vibration (accepted).

8. Happawana, G.S. and Azene, M., "Forced Vibration Amplitude of Mistuned Blades", (in preparation).

Personnel

Two faculty members and one graduate student at Purdue University and one faculty member at Southern University were partially funded by this Contract:

FACULTY: Southern University: Professor Muleneh Azene
           Purdue University: Professor Osita D.I. Nwokah
                                     Professor Anil K. Bajaj

GRADUATE STUDENT: Gemunu S. Happawana (Doctoral Student)

Mr. G.S. Happawana completed his doctoral thesis *"Structural Analysis of Mistuned Spatially*

Periodic Systems: A Singular Perturbation Approach" in August 1994 from Purdue University. Professor *Osita D.I. Nwokah (Resume at the back)

Presentations

Several seminar presentations resulted from this work. Details are contained in the individual professors resumes.

ABSTRACT

Structural systems with spatial symmetry are very often encountered in engineering practice. Typical examples of such systems include space platforms, multispan flexible continuous bridge structures, gas turbine engines, large space circular antennas and floppy disks in memory devices. These systems are usually analyzed by assuming some structural uniformity, often referred to in the literature as a "tuned" system condition. However, small nonuniformities inherent from practical realities that invariably arise due to manufacturing and material tolerances preclude the existence of such uniformity. These small nonuniformities or variations present in system parameters are commonly referred to as "structural mistuning". It has received wide attention in the vibration literature because of the fact that a small amount of mistuning under appropriate conditions can cause unexpectedly large amplitudes of vibration compared to those predicted on the basis of a perfectly tuned system.

In this final report, the dynamics of mistuned cyclic systems with special reference to strongly coupled bladed disk assemblies has been investigated. The analysis has utilized ideas from group representation theory, bifurcation theory, singular perturbation theory and modal analysis techniques. The general analysis methodology developed herein is applicable to any disk to which is attached a set of n blades which are strongly coupled cyclically, and mistuning or variations can arise in any of the system parameters. In particular, this study provides qualitative and, more importantly, quantitative information in the form of uniformly valid asymptotic expansions for the eigenfrequencies and the modal vectors of the structure. These expansions are used to describe the phenomena of eigenvalue curve veering, modal rotations, and other manifestations of the sensitive dependence of eigenfunctions on system parameters which lead to modal bifurcations in the forced response of mistuned cyclic systems. Since this approach is general and systematic, the methodology developed here is also extended and applied to other discrete and continuous structures as well.